Chapter 1

Introduction and summary

This chapter serves as an introduction to the research questions and results on double-diffusive convection due to sidewall heating or cooling as presented in chapters 2 – 6 of this thesis. An example of a numerical simulation of double-diffusive convection is presented, together with a description of the mechanism of layer formation (1.1). A review of the modern literature on experimental and numerical models of double-diffusive convection due to lateral forcing then follows (1.2). Based on this review the research questions are posed that are addressed in the subsequent chapters. The contents of these chapters are briefly summarized (1.3).

1.1 Motivation

Density gradients due to temperature and salinity differences are for a large part responsible for convective transports of heat, salt and other constituents in the ocean. Some decades ago, it was assumed that these transports primarily took place in an environment dominated by large-scale turbulence resulting in locally well mixed distributions. As a result of this assumption, transport was modelled through eddy mixing coefficients relating fluxes of constituents to the mean smoothed gradients, and the measurements available were interpreted as to fit into these models in a continuous way. The availability of an increasing amount of measurements on a smaller scale, however, lead to problems with the qualitative interpretation of the data in terms of the model assumptions [*Turner*, 1981].

Nowadays, with the advent of measuring instruments that are able to record salinity and temperature profiles on centimeter scales, it has become clear that the distribution of these properties is generally not smooth; small scale (i.e. tenths to hundreds of metres) layered structures exist

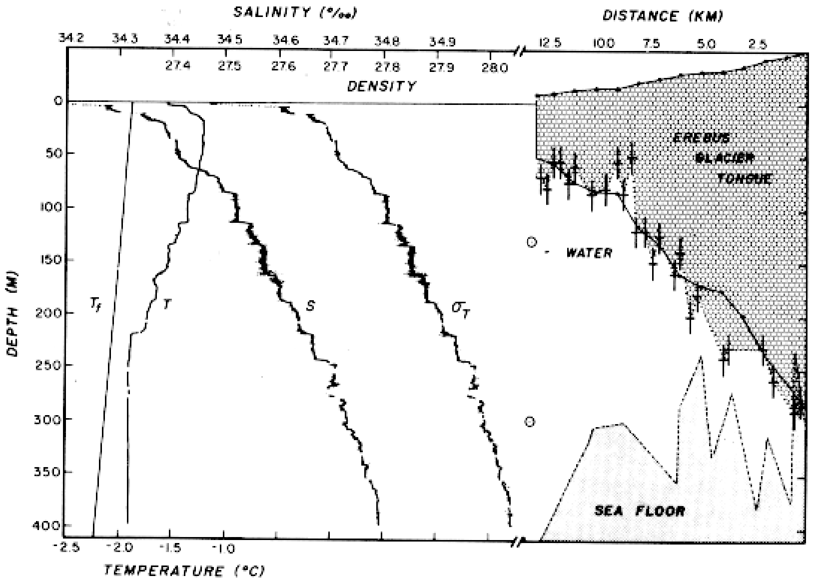

Figure 1.1: *Vertical temperature (T), salinity (S) and density (σ_T) distributions as measured about 200 meters from the upper edge of Erebus Glacier Tongue, Antarctica (from: Jacobs et al. [1981]). The profiles reveal a layered structure in the upper 300 metres of the ocean.*

in several parts of the ocean in which well mixed regions are separated by interfaces in which steep vertical property gradients are present. Often, these structures result from perturbations of a stable salt-stratified ocean by thermal gradients.

The investigations in this thesis are motivated by the recordings of layered structures in a stably stratified polar ocean in the vicinity of icebergs. In such an environment, a stable stratification in the ocean set up by a stabilizing salinity gradient is laterally cooled by the edge of an iceberg. The structures have been reported in several regions of both the northern part of the Antlantic and around the Antarctic. As an example, Fig. 1.1 shows the vertical temperature, salinity and density distributions as recorded by *Jacobs et al.* [1981] near Erebus Glacier Tongue in the Antarctic. The figure reveals a structure consisting of well-mixed horizontal layers with an average thickness of 17 metres, separated by thin diffusive interfaces in which steep temperature and salinity gradients exist. In the region where layers are present the overall distribution of heat and salt is stable, which indicates that lateral cooling of the ocean by the tongue is responsible for the layer formation, not vertical cooling at the top due to a cold atmosphere. The observed layer thickness is of the same order as would be expected from physical arguments discussed below.

The transport of heat and salt induced by these structures is expected to be quite different from the transport that is induced by thermal convection alone, and will both influence the local transports as well as the way in which the iceberg melts. This motivates a deeper study on the origins of layer formation and both the qualitative and quantitative aspects of the layer formation process.

The convection that is generated in stably stratified liquids by the opposing buoyancy effects of temperature and salinity (or a couple of different fluid constituents with this property) and their different molecular diffusivities is known as *double-diffusive* convection. Different types of motion exist depending on whether the stable stratification is provided by the component with the lowest or the highest molecular diffusivity.[1] Furthermore, the direction of the gradients heavily influences the evolution of the flow.

In this thesis we restrict ourselves to the case of a stable salt-stratified, initially motionless water column in a container to which a lateral temperature gradient is applied by heating or cooling (one of) the sidewalls. When the temperature difference between a vertical wall and the liquid exceeds a critical value, a layered system develops which consists of horizontal convection cells separated by diffusive interfaces. At this point a picture of the layer formation process is helpful. Fig. 1.2 shows the simulation of layer formation in a square cavity of 20 cm height containing a liquid which was initially motionless and linearly stratified with salt. The salinity gradient is maintained by prescribing a constant salinity difference between the horizontal walls. A lateral temperature difference far beyond the critical value is applied by heating the left boundary of the cavity with a constant value for the temperature. The sequence of pictures shows the formation of a series of layers extending horizontally into the bulk, separated by sharp interfaces. Several features of the layer formation in Fig. 1.2, like the characteristic thickness of the layers, the mixing properties and the merging of layers observed at several stages, are subject of investigation in this thesis.

The structure of the different dependent quantities in the layers in Fig. 1.2f is revealed in Fig. 1.3. In the convective layers salt is well mixed and the temperature is stably stratified. In the interface between the layers the temperature distribution is unstable and a large shear exists, but the very strong salinity gradient prevents shear instability to occur.

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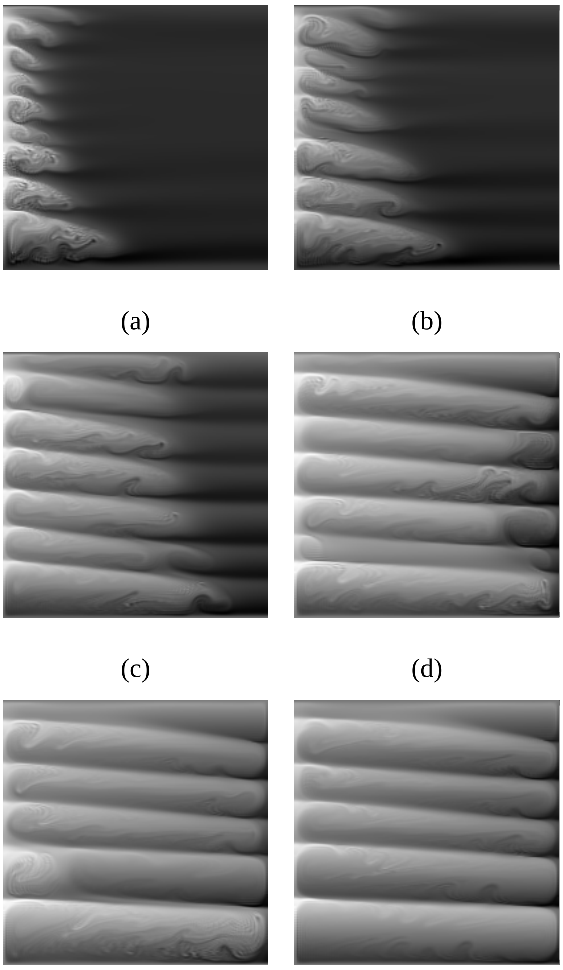
[1] If the stratification is provided by the component with the lower molecular diffusivity, the stratification is of *diffusive* type, otherwise it is of *finger* type. Although finger type systems are important in oceanography [*Schmitt*, 1994], they are not investigated in this thesis.



(a) (b)

(c) (d)

(e) (f)

Figure 1.2: *Development in time of the double-diffusive intrusions in a container of 20 cm height. At the left is the heated sidewall. Shown is the salinity distribution minus the initial linear stratification; white stands for high salinity with respect to the initial stratification while black indicates a relatively low salinity. The pictures are taken successively at 50 min. (a), 83 min. (b), 167 min. (c), 333 min (d), 500 min. (e) and 667 min. (f).*

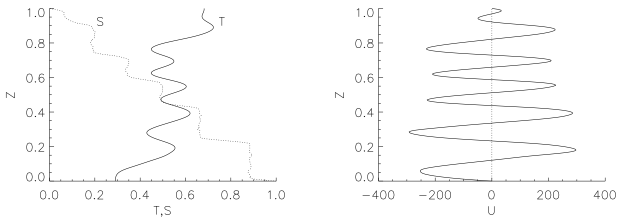


Figure 1.3: *The distribution of temperature T, salinity S and the horizontal component of the velocity U along a vertical section through the centre of the cavity for the flow as depicted in Fig. 1.2f.*

A simple physical argument for the thickness of the layers is available by considering a fluid parcel near the hot left sidewall before layer formation has started [*Chen et al.*, 1971]. Due to the sidewall heating the buoyancy of this parcel is increased and thus it rises in a vertical buoyancy driven boundary layer. Because of the low solutal molecular diffusivity the parcel remains essentially at a constant salinity as it continues to rise. The stable background stratification causes the parcel to become denser relative to its surroundings, and finally it reaches the same density as the bulk. If viscous shear forces are overcome the parcel cannot rise anymore and is forced to move laterally; double-diffusive instability results in the formation of convection cells near the heated wall which eventually evolve into the observed layers. Thus, the lengthscale over which a parcel can rise due to heating depends both on the strength of the salinity gradient and the temperature difference between the wall and the bulk of the fluid; this scale is given by

$$\eta = \frac{\alpha \Delta T}{\beta \phi_0}, \tag{1.1}$$

where ΔT is the laterally imposed temperature difference, ϕ_0 is the strength of the initial stable salinity gradient and α, β are the thermal and solutal expansion coefficients, respectively.

The observed tilt of the layers is a feature typical of double-diffusive convection; the heated fluid that is transported away from the wall loses its heat to the cooler return flow in the layer just above it through diffusion much faster than its salt, and therefore becomes heavier and tends to sink. We note that there is no qualitative difference between models that use sidewall heating and those in which sidewall cooling is applied; only the orientation of the velocity vectors and the tilt of the layers are reversed.

1.2 A review of previous work

In the past decades double-diffusive convection has developed into a separate area of research, with results available from observations, experiments and both analytical and numerical models [*Fernando and Brandt*, 1995]. In this section we present an overview of the previous work that is relevant to the research as described in this thesis. Although the motivation for these investigations often stems from oceanography, and indeed the development of the subject has been initiated in this area [*Stern*, 1960; *Stommel et al.*, 1956], double-diffusive convection has been recognised important in other areas as well. In particular, we mention its relevance for the modeling of solar ponds [*Akbarzadeh and Manins*, 1988] and magma chambers [*Fernando and Brandt*, 1995].

We start the survey with the lateral heating experiments by *Thorpe et al.* [1969]; they investigated the formation of convection cells in a salt-stratified fluid contained in a narrow, laterally heated vertical slot. Convection cells were reported along the heated wall. They suggested an instability of the thermal boundary layer to be responsible for the cell generation based on the presence of horizontal temperature and salinity gradients in the boundary layer. Furthermore they suggested that the difference in diffusivities of heat and salinity plays a central role in the built-up of the horizontal gradients. Their description thus contains the key elements of a double-diffusive mechanism, and they related it to the mechanism of layer formation in the ocean.

A criterion for the onset of layer formation and the corresponding layer size in stratified containers was established in experimental and numerical studies [*Chen et al.*, 1971; *Wirtz et al.*, 1972]. A critical Rayleigh number existed above which simultaneous layer formation occurs along the entire laterally heated wall as an instability of the thermal boundary layer. Based on a series of experiments the layer thickness was determined to be equal to the potential rise of a heated element in a stratified liquid according to the mechanism described in the introduction. At a Rayleigh number below critical successive layer formation occurred, which is induced by the horizontal boundaries and extends over only a limited vertical distance in the container.

Theoretical studies consider the onset of instability of a parallel basic flow in very simple (semi-) infinite geometries. *Thangam et al.* [1981] investigated the instability of a basic parallel flow in a differentially heated narrow (vertically infinite) slot in which a stable vertical salt gradient is present. In this case a time-independent background flow exists which can be calculated analytically. Their objective was to investigate the transition from shear instabilities (in case of a weak salt gradient) to double-diffusive instabilities for stronger salinity gradients. They found a transition from stationary steady shear instability for low weak salinity gradients, through an oscillatory instability, to steady double-diffusive instability valid for increasingly stronger salt gradients.

The stability analysis in a wide container (which is modelled by sidewall heating of a fluid which extends infinitely into the vertical and is infinite in the horizontal direction to the right) is more complex since the background flow in such a configuration is time-dependent. The stability analysis of *Kerr* [1989] and *Kerr* [1990] revealed the existence of an oscillatory instability and subcritical finite-amplitude (but unstable) flow.

Considering the available results on double-diffusive layer formation, *Huppert and Turner* [1978] suggested that the transport of meltwater from melting icebergs in polar seas is strongly influenced by the presence of double-diffusive layers due to a stable salt gradient in the ocean. In a simple experiment using a block of ice which melts in a column of salt-stratified water they showed that the fresh meltwater is mixed into the double-diffusive layers and transported mainly laterally, instead of being transported vertically along the ice edge as would be the case if the salt gradient was absent. In a profound study [*Huppert and Turner*, 1980] the layer formation was investigated for a large range of Rayleigh number using blocks of ice and both heated and cooled cylinders. The lengthscale η (1.1) was shown to be valid even for very large Rayleigh number, and transport of meltwater is mainly lateral as was suggested. Furthermore, this study revealed that the meltwater has little influence on the layer structure.

The previous investigations have been concerned mainly with an analysis of the double-diffusive instabilities which are responsible for the formation of a layered structure and the layer thickness of the developed layers. The experimental investigations of *Tanny and Tsinober* [1988] form a thorough study on the evolution of double-diffusive layers in salt stratified wide containers which are heated from one side using a prescribed time-dependent exponential temperature profile. They showed that the stability diagram for the wide container is essentially the same as for the narrow slot. The layer formation process appears to be largely independent of the temperature profile described. After layers have formed they tend to merge in, as they described it, a ”chaotic” way, i.e. no specific lengthscale for the layer thickness can be defined, until they finally attain a thickness of order η in correspondence with the results of *Chen et al.* [1971]. An investigation of the vertical density distribution across the layer interfaces revealed that layers do not merge as a result of density equalisation; they proposed interface breakdown and interface migration as the dominant mechanisms behind merging.

The presence of a linearly unstable temperature stratification in addition to the stable salt stratification - but with the total density distribution being stable - is interesting from both oceanographical and theoretical views. The presence of an unstable vertical temperature gradient in the ocean is very common as a result of cold, fresh water being on top of warmer but saltier water; examples are the cooling of polar seas by a cold atmosphere, and the outflow of warm, saline Meditterranean water in the relatively cold and fresh Atlantic Ocean near Gibraltar. In a double-diffusive context the unstable thermal stratification is interesting as a source of potential energy which may be converted into kinetic energy when the total density distribution becomes unstable due to a weakening of the salinity gradient. This energy conversion is a possible mechanism behind the so called ”self-propagation” of layers, i.e. the continuous propagation of the layer fronts even after sidewall heating has stopped.

The doubly stratified experiments by *Jeevaraj and Imberger* [1991] in laterally heated wide containers showed that the stability characteristics of a doubly stratified system are comparable to a singly stratified model, with only minor changes in critical Rayleigh number and layer thickness, but with convection more vigorous when the unstable temperature stratification becomes more important. After sidewall heating was stopped, self-propagation of layers was not observed.

Schladow et al. [1992] were able to classify the flows, depending both on the relative strength of the thermal stratification compared to the saline stratification (i.e. the vertical gravitational stability) and the lateral heatflux at the heated wall (the lateral stability). Three classes were identified; the first was identified with large lateral stability and large gravitational stability (a situation comparable with singly stratified experiments) and showed merging as a result of horizontal motions induced by the intrusions. The second class, corresponding with lower gravitational stability showed a more vigorous convection and merging was observed as a breakdown of the interfaces near the heated wall due to horizontal motions. In the third class, corresponding to low gravitational and lateral stability, self-propagation was observed after sidewall heating was removed.

The numerical work on double-diffusive systems has for the largest part concentrated on narrow container configurations in which a stable salt gradient is laterally heated from the side through one or both sidewalls. The development of layers in a container in the neighbourhood of the critical Rayleigh number was investigated by *Lee and Hyun* [1991] and they retrieved basically the same results as *Chen et al.* [1971]; for supercritical simulation layers were formed simultaneously with a thickness of order η, while in subcritical cases layer formation was successive. Since a fixed temperature difference was prescribed but saline forcing was absent, the salinity gradient became eroded and finally only the thermally driven single-cell pattern remained. In order to study the long-term behaviour of such a double-diffusive system, non-trivial steady states may be traced by fixing both the temperature at the sidewalls and the salinity at the horizontal walls. In such a configuration, *Lee et al.* [1990] showed that four different flow regimes are possible depending on the strength of the thermal and saline forcing. In addition to the simultaneous and successive regimes, a stagnant flow regime exists when the saline forcing dominates the thermal forcing, while a unicellular flow regime is present in case the thermal forcing is much larger than the saline forcing. These findings were supported by accompanying experiments [*Lee and Hyun*, 1991], which also show that the approach to the final state identifying the flow regime is not trivial; for example, in the unicellular flow regime the corresponding flow pattern is reached after subsequent merging of cells that are a result of double-diffusive instabilities.

In the latter configuration, the existence of different flow regimes encourages the investigation of the steady state structure of laterally heated narrow container configurations. The numerical study of *Tsitverblit and Kit* [1993] showed that multiple steady states exist in the double-diffusive regime due to the interplay of heat and salt. They suggested a relationship between the multiplicity of solutions and chaotic merging of layers as observed in the experiments of *Tanny and Tsinober* [1988], although the lack of a stability analysis and the differences in geometry and parameter range between the numerical model and the experiments did not improve their claim. An extended analysis of the configuration is presented in *Tsitverblit* [1995] for a larger range of parameters, showing an increasing complexity and multiplicity of solutions for larger solutal Rayleigh numbers.

1.3 Overview of the thesis

Main problem in this thesis is to determine the large scale effects that double-diffusive layered structures have on the vertical transport of fluid constituents. Therefore, our main goal is to derive effective diffusivities for heat and salt from our model results. In order to achieve this goal we have adopted the following approach. The dynamics and physics of double-diffusive layered structures due to lateral forcing are investigated in Chapters 2 – 4. In Chapter 5, the conditions for existence of layer formation and the induced transports in a simple ice-plate geometry are considered. Finally, in Chapter 6, effective vertical diffusivities for salinity are estimated.

The availability of both experimental and numerical results on the narrow container configurations using moderate values for the forcing parameters enables us to start with the analysis of these type of systems. In *Kranenborg and Dijkstra* [1995] the results of *Tsitverblit and Kit* [1993] were recomputed and extended. They showed that, studying the flow evolution towards a single, thermally dominated state in the supercritical, unicellular regime, the multiplicity of solutions was reduced, leaving only the thermally dominated unicellular solution linearly stable. Still, the multicell unstable solutions were shown to be physically relevant since during the approach to the single stable state the flow remained for a very long time close to the multicell state. These results suggest that the flow regimes found by *Lee et al.* [1990] correspond with stable states of the steady equations, while the many unstable states play a role as attractors which determine the approach towards the final, stable state. Thus it is straightforward to suggest that the boundary between two flow regimes corresponds with a stable state becoming unstable due to a singularity, and therefore we pose as a research problem for Chapter 2:

- *In which way are boundaries between the different flow regimes related to the underlying dynamical structure of the system?*

In order to answer this question the long time behaviour of flows in the narrow slot configuration as used by *Tsitverblit and Kit* [1993] and *Lee et al.* [1990] is investigated; this investigation extends the results of *Kranenborg and Dijkstra* [1995]. As in the latter study, continuation methods (see Appendix A) are applied to trace the branches of steady solutions in parameter space, while the corresponding linear stability is calculated simultaneously. The resulting structure of steady solutions reveals that boundaries between the different flow regimes as reported by *Lee et al.* [1990] are, to some extent, related to paths of bifurcation points in parameter space; the boundary between the thermally dominated unicellular flow and the double-diffusive multicell flow is determined as the path of bifurcation points in parameter space on which the unicellular pattern becomes unstable. The exact location of this boundary appears difficult to determine. Even in the unicellular regime signs of double-diffusion are present, as was already shown in *Kranenborg and Dijkstra* [1995], by the rapid evolution from a four cell pattern, through the two cell pattern corresponding with the unstable steady state, towards the stable unicellular thermally dominated pattern. In Chapter 2 it is shown through accurate calculations that the transition from the unstable two-cell pattern towards the unicellular pattern takes place through an instability as predicted by the unstable eigenvector only after a very long preconditioning phase. Here the mechanism behind the transition is interface migration. Other boundaries could not be clearly established due to an abundance of singularities occuring in the double-diffusive regime.

In Chapter 3 the evolution of the double-diffusive layers is considered. In the experimental studies of *Tanny and Tsinober* [1988]; *Jeevaraj and Imberger* [1991] and *Schladow et al.* [1992] the evolution of these layers is characterised by the occurrence of layer merging. Since layer merging increases the final scale of the layers and thereby the mixing characteristics of the flow, it is worthwhile to investigate the physical mechanisms behind it and, if possible, to relate it to the dynamical picture presented in Chapter 2. Thus, the question for Chapter 3 becomes:

- *What are the physical mechanisms behind layer merging during layer evolution?*

The time-dependent evolution of the double-diffusive instabilities is studied in the same range of Rayleigh number as the experiments of *Tanny and Tsinober* [1988] and *Jeevaraj and Imberger* [1991]. The layer scale of the simulated flows agrees very well with the various experimental results [*Chen et al.*, 1971; *Tanny and Tsinober*, 1988; *Jeevaraj and Imberger*, 1991]. The simulations on a high-resolution grid allow a detailed analysis of the layer merging process; both shear instabilities and density equalisation are ruled out as relevant. Instead, two instability mechanisms are proposed; an instability leading to layer migration as found in Chapter 2 and a differential entrainment mechanism based on a local analysis of the interface Richardson number.

If a destabilizing initial unstable temperature distribution is present in addition to a stabilizing salt gradient, and if the temperature gradient is large, then layers may continue to propagate into the bulk of the fluid even after the sidewall heating has been turned off. This behaviour has only been observed in a few experiments [*Schladow et al.*, 1992] but an analysis of the physical mechanism is not yet available. In Chapter 4, we therefore pose the following question:

- *What is the physical mechanism behind self-propagation of layers?*

In a doubly stratified system in which the sidewall heating is turned off, self-propagation of layers is shown to exist in a wide container. This is in contrast with the corresponding singly stratified system where self-propagation does not appear. Excessive transport of salt along the heated wall and intense vertical convection results in a heavy patch of fluid located near the heated wall. After sidewall forcing is stopped the patch of fluid adjusts itself to a neutrally buoyant level, which gives rise to a background flow in which local instabilities may occur. This background flow thus generates the propagation of the layers.

In an oceanographic context, vertical ice boundaries (for example ice slabs or icebergs) can provide the lateral cooling of a stably stratified fluid. The laboratory experiments of *Huppert and Turner* [1980] show that next to an ice block double-diffusive layers are formed with lengthscale η. In Chapter 5 we investigate the following questions:

- *What are the conditions for existence of layer formation near an ice plate, and how is the transport of heat and salt altered by the double-diffusive flow in such a geometry?*

A cooled solid slab is used as a simple model of an ice plate. It is shown that the thickness of the slab must be larger than the lengthscale η for layers to exist. At the lower slab edge a buoyancy jump develops which isolates the double-diffusive flows next to the slab from other fluid regions; the layer formation next to the slab takes place in the same way as in the cavity simulations of Chapters 3 and 4. The buoyancy jump causes a strong decrease of the vertical heat and salt transport, compared to the transports caused by thermal gradients only.

Finally, in Chapter 6 the effective vertical diffusivity for salt is estimated from a series of high-resolution simulations. An effective vertical thermal diffusivity could not be determined due to the absence of a vertical background temperature gradient. The vertical salt fluxes over the diffusive interfaces are shown to satisfy a well-known flux law. From these fluxes an estimate of the vertical salt diffusivity is determined for the parameter range corresponding to the simulations. Next, the estimate of the salt diffusivity is extrapolated towards oceanographic conditions, yielding a value of the same order as those determined from measurements. This result shows that some of the oceanic layered structures may be generated by double-diffusive instabilities due to lateral temperature gradients.

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